Pergamon

Int. J. Multiphase Flow Vol. 22, No. 1, pp. 177–184, 1996
Copyright © 1996 Elsevier Science Ltd
Printed in Great Britain. All rights reserved
0301-9322/96 $15.00 + 0.00

0301-9322(95)00062-3

ON THE INERTIA EFFECT IN EDDY INTERACTION MODELS

D. I. GRAHAM

School of Mathematics and Statistics, University of Plymouth, Plymouth PL4 8AA, England

(Received 26 April 1994; in revised form 1 February 1995)

Abstract—It has been demonstrated experimentally and theoretically that the dispersion coefficients of non-fluid particles in isotropic turbulent flows can exceed the dispersion coefficient of fluid particles. This "inertia effect" cannot be allowed for in eddy interaction models in which it has been shown that the long-time dispersion coefficient of non-fluid particles is always less than that of fluid particles. In the limit of infinitely heavy particles, the difference between the dispersion coefficients is due to the difference between the Lagrangian and Eulerian integral time scales τ_L and τ_E and it can be shown that $\tau_E < \tau_L$ in existent eddy interaction models. In this paper, the relationship between τ_E and τ_L is investigated for a modified eddy interaction model for which it is possible that $\tau_E > \tau_L$, which will lead to greater long-time dispersion of heavy non-fluid particles than of fluid particles. Numerical simulations are carried out to quantify the effect of the modifications on the dispersion of finite size particles.

Key Words: particle dispersion, turbulent flow, inertia effect, Eulerian time scales, dispersion coefficients

1. INTRODUCTION

The eddy interaction model for particle dispersion in turbulent flows was first used by Hutchinson *et al.* (1971) to model unidirectional particle dispersion in turbulent pipe flows. Several authors (Gosman & Ioannides 1981; Shuen *et al.* 1983, for example) have since developed the method, enabling its use in more complex turbulent flows. The model uses a stochastic approach to predict the characteristics of the discrete phase in dispersed two-phase flows. Each of a number of individual particles is tracked through a series of interactions with fluid eddies whose length, "lifetime" and velocity can all be random variables. The "crossing trajectory effect" (Yudine 1959), whereby particle dispersion is reduced in the presence of strong body forces due to particles rapidly passing through eddies, is allowed for by specifying that a new eddy be entered once the distance between a particle and the centre of an individual eddy exceeds the eddy length.

In the model, the finite eddy length also influences the "inertia effect". Graham & James (1996) showed that, under the assumption that the drag on non-fluid particles is Stokesian, the eddy interaction model always predicts that heavy non-fluid particles will be dispersed less rapidly, in the long-time limit, than fluid particles. This was demonstrated to be the case whether or not random eddy lifetimes and eddy lengths were used in addition to the usual random eddy velocities. However, as discussed below, there is evidence that the dispersion coefficient for very heavy particles should in fact exceed that for fluid particles. The purpose of this paper is to investigate the possibility that eddy interaction models can be modified to predict the enhanced dispersion of non-fluid particles by comparing Eulerian and Lagrangian fluid velocity auto-correlations and integral time scales for a modified eddy interaction model.

It can be shown that, in homogeneous isotropic and stationary turbulence in which the turbulence intensity is equal to u', and assuming that Stokesian drag is the only force acting on a particle, the long-time dispersion coefficient of non-fluid particles is given by

$$\bar{D}_p = u'^2 \int_0^\infty R_L^p(\tau)\,d\tau = u'^2 \tau_L^p \qquad [1]$$

where $R_L^p(\tau)$ is the fluid velocity auto-correlation following the path of a non-fluid particle and τ_L^p is the corresponding integral time scale of the fluid motion. In the limiting case of infinitely heavy particles, $R_f^p(\tau)$ is a correlation of velocities at a fixed point, i.e. an Eulerian fluid velocity auto-correlation. In this case, τ_f^p is equal to the Eulerian integral time scale, τ_E. The dispersion of very heavy particles is thus dependent upon this Eulerian integral time scale. In the limiting case of fluid particles, $R_f^p(\tau)$ is a correlation of velocities following fluid particles, i.e. the Lagrangian auto-correlation. In this case, τ_f^p is equal to the Lagrangian integral time scale, τ_L.

Reeks (1977) and Pismen & Nir (1978) have developed theories which predict that very heavy particles disperse more rapidly, in the long term, than fluid particles. The experiments of Wells & Stock (1983), the direct numerical simulations of Squires & Eaton 1991 and the large eddy simulations of Deutsch (1992) (reported by Minier & Pozorski 1992), all indicate the possibility that the dispersion coefficient for heavy particles exceeds that for fluid particles. It is therefore important that models used to simulate dispersion of non-fluid particles are flexible enough to allow for this inertia effect. It is the question of the flexibility of the eddy interaction model which is addressed here.

2. ANALYSIS OF EDDY INTERACTION MODELS

It is clear from the analysis of Graham & James (1996) that the decreased dispersivity of heavy particles is due to the constraint used within the conventional eddy models that non-fluid particles interact with a given eddy for a time not exceeding what has come to be called the eddy lifetime. It should, however, be noted that this "eddy lifetime" is in fact the interaction time for a fluid particle. In the following discussion, the term "eddy lifetime" is avoided in favour of the term "fluid particle interaction time", which is denoted as T_f. Within current eddy interaction models, the interaction time for non-fluid particles is often less than T_f, since the particle has frequently crossed the eddy within T_f. The modified eddy interaction model considered below allows the interaction time to exceed T_f and the relationship between Lagrangian and Eulerian integral scales is determined for the new method.

For simplicity, the analysis of the models is restricted to one spatial dimension. The constant fluid particle interaction time T_f is chosen to be twice the Lagrangian integral time scale of the turbulence τ_L. Choosing T_f in this way ensures that the Lagrangian integral time scales of the actual and model turbulence are equal. This ensures that the dispersion coefficient of fluid particles is predicted correctly. The constant eddy length L_e is chosen to be twice the Eulerian longitudinal integral length scale, Λ_E. This choice of eddy length ensures that the longitudinal length scales of the actual and model turbulence are equal. The reader is referred to Graham & James (1996) for a more complete discussion of length and time scales in eddy interaction models.

2.1. *Eddy interaction model specifications*

In this section, a modification to the eddy interaction model is proposed. Present eddy interaction models assume that the eddy interaction time is determined as the minimum of T_f and the eddy "crossing time" (see Gosman & Ioannides 1981). The modified method assumes that the maximum interaction time is equal to some value T_{max}, independently of T_f. The conventional eddy interaction model is retrieved by setting $T_{max} = T_f$. The eddy interaction time is determined by

(1) if $|u_r| < L_e/\tau_r$, particle is captured, set $t_i = T_f = 2\tau_L$
(2) otherwise, $t_c = -\tau_r \log(1 - L_e/|u_r|\tau_r)$, and $t_i = \min(T_{max}, t_c)$.

In the above, u_r is the fluid–particle relative velocity at the beginning of an eddy interaction, τ_r is the particle relaxation time, L_e is the eddy length and τ_L is the Lagrangian integral time scale of the simulated turbulence.

2.2. *Eulerian time auto-correlations*

In order to proceed with the analysis, we first note that the fluid velocity auto-correlation along a non-fluid particle path can be found using the method of Wang & Stock (1992). The interaction time t_i can be considered to be a random variable. Graham & James (1996) developed the ideas of Wang & Stock (1992) to determine $R_f^p(\tau)$ from eddy interaction models. Using this analysis, it can be shown that the auto-correlation function is given by

$$R_f^p(\tau) = \begin{cases} \dfrac{\displaystyle\int_{|\tau|}^{T_{max}} \int_0^{u_{max}(L_e,t')} 2h(u_r)\,du_r\,dt'}{\displaystyle\int_0^{T_{max}} \int_0^{u_{max}(L_e,t')} 2h(u_r)\,du_r\,dt'} & |\tau| \leqslant T_{max} \\ 0 & \text{otherwise} \end{cases} \qquad [2]$$

In [2], $h(u_r)$ is the p.d.f. of the fluid–particle relative velocity u_r (equal to the fluid velocity in the case of an infinitely heavy particle, i.e. a fixed point) and $u_{max}(L_e, t')$ is the maximum relative fluid–non-fluid velocity such that the time taken for the non-fluid particle to cross an eddy of length L_e is greater than t' (see Graham & James 1996),

$$u_{max}(L_e, t') = \frac{L_e}{\tau_r(1 - e^{-t'/\tau_r})}. \qquad [3]$$

Eulerian (fixed-point) correlations are determined by allowing τ_r to approach infinity, so that $u_{max}(L_e, t') \to L_e/t'$. In this case, the Eulerian time correlation is given by

$$R_E(\tau) = \begin{cases} \dfrac{\displaystyle\int_{|\tau|}^{T_{max}} \int_0^{L_e/t'} 2h(u_r)\,du_r\,dt'}{\displaystyle\int_0^{T_{max}} \int_0^{L_e/t'} 2h(u_r)\,du_r\,dt'} & |\tau| \leqslant T_{max} \\ 0 & \text{otherwise} \end{cases} \qquad [4]$$

As noted above, the relative velocity u_r is equal, in the case of a fixed point, to the fluid velocity u_f, so that $h(u_r)$ is then the p.d.f. of the fluid velocity $h(u_f)$. The most common fluid velocity distribution used in the literature (Gosman & Ioannides 1981; Shuen *et al.* 1983; Govan *et al.* 1989, is the normal distribution with mean zero and standard deviation u'. In this case,

$$R_E(\tau) = \begin{cases} \dfrac{\displaystyle\int_{L_e/T_{max}}^{L_e/|\tau|} \frac{L_e}{u_f} e^{-1/2(u_f/u')^2}\,du_f + T_{max}\,\mathrm{erf}\left(\frac{\sqrt{2}}{2}\frac{L_e}{T_{max}u'}\right) - |\tau|\,\mathrm{erf}\left(\frac{\sqrt{2}}{2}\frac{L_e}{|\tau|u'}\right)}{\displaystyle\int_{L_e/T_{max}}^{\infty} \frac{L_e}{u_f} e^{-1/2(u_f/u')^2}\,du_f + T_{max}\,\mathrm{erf}\left(\frac{\sqrt{2}}{2}\frac{L_e}{T_{max}u'}\right)}, & |\tau| \leqslant T_{max} \\ 0 & \text{otherwise} \end{cases} \qquad [5]$$

The Eulerian temporal auto-correlation function $R_E(\tau)$ therefore depends upon the eddy length L_e and the maximum interaction time T_{max} and is independent of the fluid particle interaction time T_f. Equation [5] has been evaluated using 64-point Gauss–Legendre numerical integration for three different values of T_{max} and for three different values of the turbulence structure parameter defined as $\beta = u'\tau_L/\Lambda_E = u'T_e/L_e$, which represents the ratio of the Lagrangian and Eulerian (longitudinal) length scales. The auto-correlations resulting from the choice of $T_{max} = \tau_L$, $2\tau_l$ and $2.8\tau_L$ for $\beta = 0.36$, 1 and 2 are illustrated in figure 1(a) (for $\beta = 0.36$), (b) (for $\beta = 1$) and (c) (for $\beta = 0.36$). The choice of constant T_f leads to the linear Lagrangian auto-correlation function illustrated in figure 1(a)–(c). It is interesting to note that, for each value of β used, the Eulerian auto-correlation resulting from use of the "standard" eddy interaction model obtained by setting T_{max} equal to the fluid particle interaction time T_f is always exceeded by the Lagrangian auto-correlation. This result is predicted by the analysis of Graham & James (1996). When $\beta T_{max}/T_f$ is small, the first terms in both the numerator and denominator of [5] become small, resulting in a linear auto-correlation function [see figure 1(c)], which is independent of the eddy length. In general, however, the auto-correlation is not linear and does depend upon L_e. With $T_{max} = 2.8\tau_L$, for all values of β, it is clear that the Eulerian integral time scale (which is the area under the Eulerian auto-correlation) is significantly greater than the Lagrangian time scale.

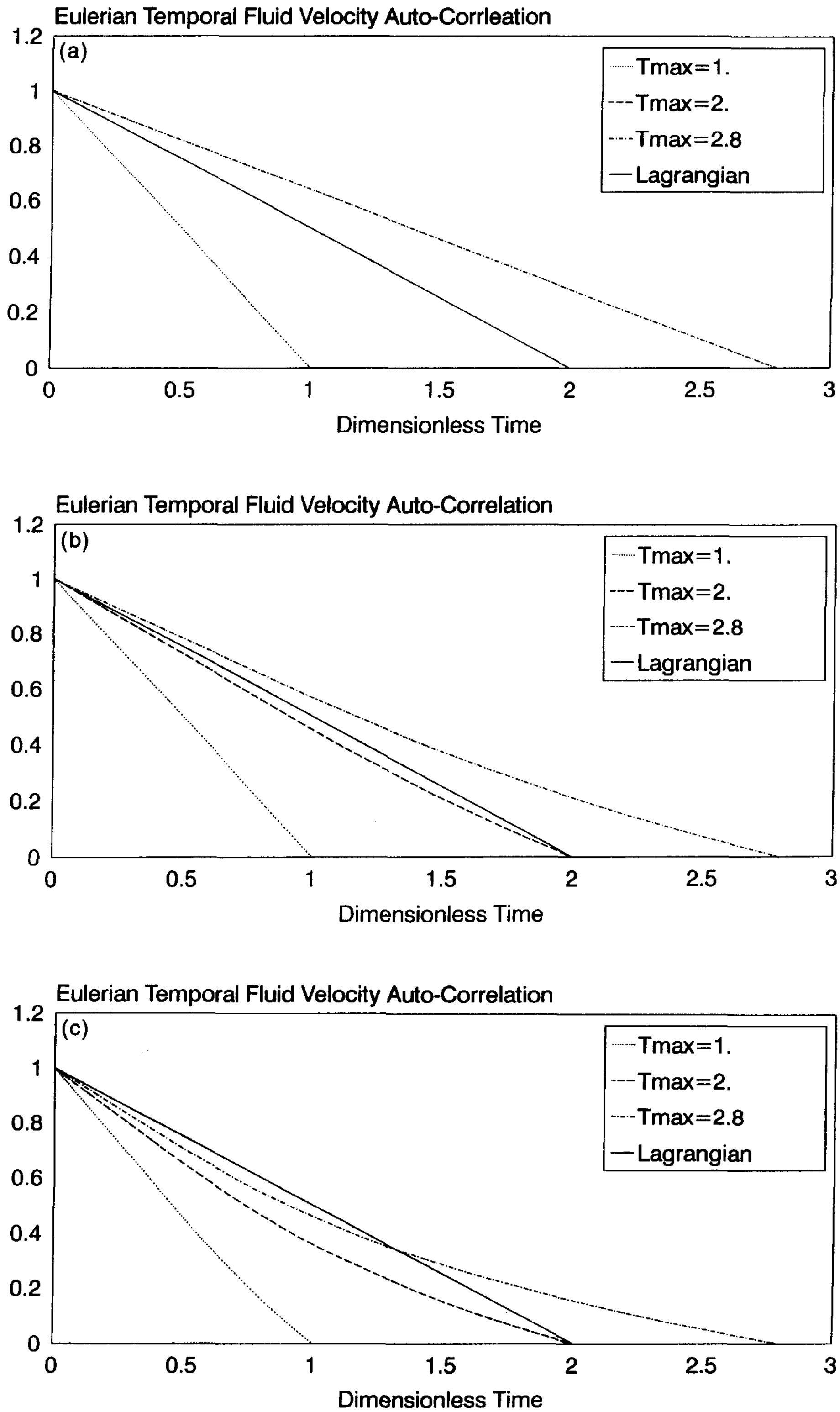

Figure 1. Eulerian temporal auto-correlation functions for various T_{max}. (a) $\beta = 0.36$, (b) $\beta = 1.0$ and (c) $\beta = 2.0$.

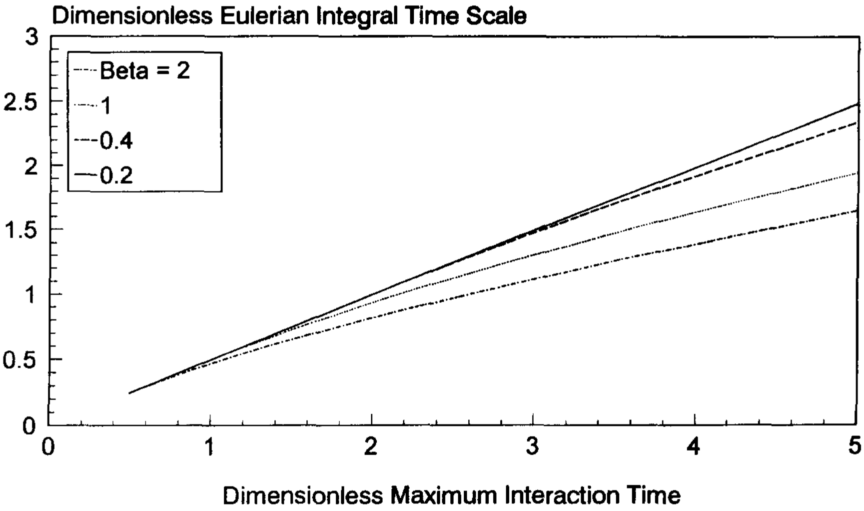

Figure 2. Dependence of Eulerian integral time scale upon maximum interaction time for various values of β.

2.3. Eulerian integral time scales

Eulerian integral time scales are determined by integration of [4]

$$\tau_E = \int_0^{T_{max}} \left[\frac{\displaystyle\int_{L_e/T_{max}}^{L_e/|\tau|} \left(\frac{L_e}{u_f} - |\tau|\right) 2h(u_f)\,du_f + (T_{max} - |\tau|) \int_0^{L_e/T_{max}} 2h(u_f)\,du_f}{\displaystyle\int_{L_e/\tau_{max}}^{\infty} \frac{L_e}{u_f} 2h(u_f)\,du_f + T\mathrm{max} \int_0^{L_e/T_{max}} 2h(u_f)\,du_f} \right] d\tau. \qquad [6]$$

Numerical integration must also be used to determine the relationship between τ_E and T_{max}. The relationship between τ_E and T_{max} is illustrated for various values of β in figure 2. When $\beta T_{max}/T_f$ is small, τ_E is equal to $T_{max}/2$, independently of the eddy length. Generally, however, since the auto-correlation is not linear and τ_E depends upon both T_{max} and L_e. The figure can be used to determine the appropriate value of T_{max} to lead to a specified ratio of Lagrangian and Eulerian time scales. Supposing that, for example, following Pismen & Nir (1978), it was required to choose T_{max} such that $\tau_E/\tau_L \approx 1.4$. Pismen & Nir's analysis leads to a value of β of approximately 0.36. From figure 2, the appropriate value of T_{max} is approximately $2.8\tau_L$. The corresponding value of T_{max}/τ_L to give $\tau_E/\tau_L = 1$ is approximately $2\tau_L$. Using the modified eddy interaction model, any required value for the ratio of Lagrangian to Eulerian time scales can be specified for any given value of the turbulence structure parameter β.

3. NUMERICAL SIMULATIONS

The analysis modified eddy interaction models in section 2 of this was concerned with infinitely massive particles, leading to fixed-point Eulerian correlations. In practice, of course, eddy interaction models are used to simulate the dispersion of particles of finite mass. It is therefore of interest to determine the influence upon the dispersion of finite particles of the modifications considered in section 2. To this end, a series of numerical simulations has been performed to evaluate the dispersion of finite particles undergoing Stokesian drag in homogeneous isotropic turbulence. The method used is similar to that used in Graham & James (1996), to which the reader is referred for further details of numerical simulations. In each case, 20,000 particles were used, the fluid velocities followed the normal distribution and the total length of the numerical simulation was 2500 τ_L. Three values of the maximum interaction time and three values of the turbulence structure parameter have been investigated. Dimensionless long-time dispersion coefficients $\bar{D}_p/u'^2\tau_L$ are plotted as functions of particle Stokes number τ_r/τ_L in figure 3(a) (for $\beta = 0.36$), (b) (for $\beta = 1$) and (c) (for $\beta = 0.36$) for $T_{max} = \tau_L$, $2\tau_L$ and $2.8\tau_L$. Figure 3(a) also illustrates the results from the analysis of Pismen & Nir (1978), which assumes a value for β equal to 0.36.

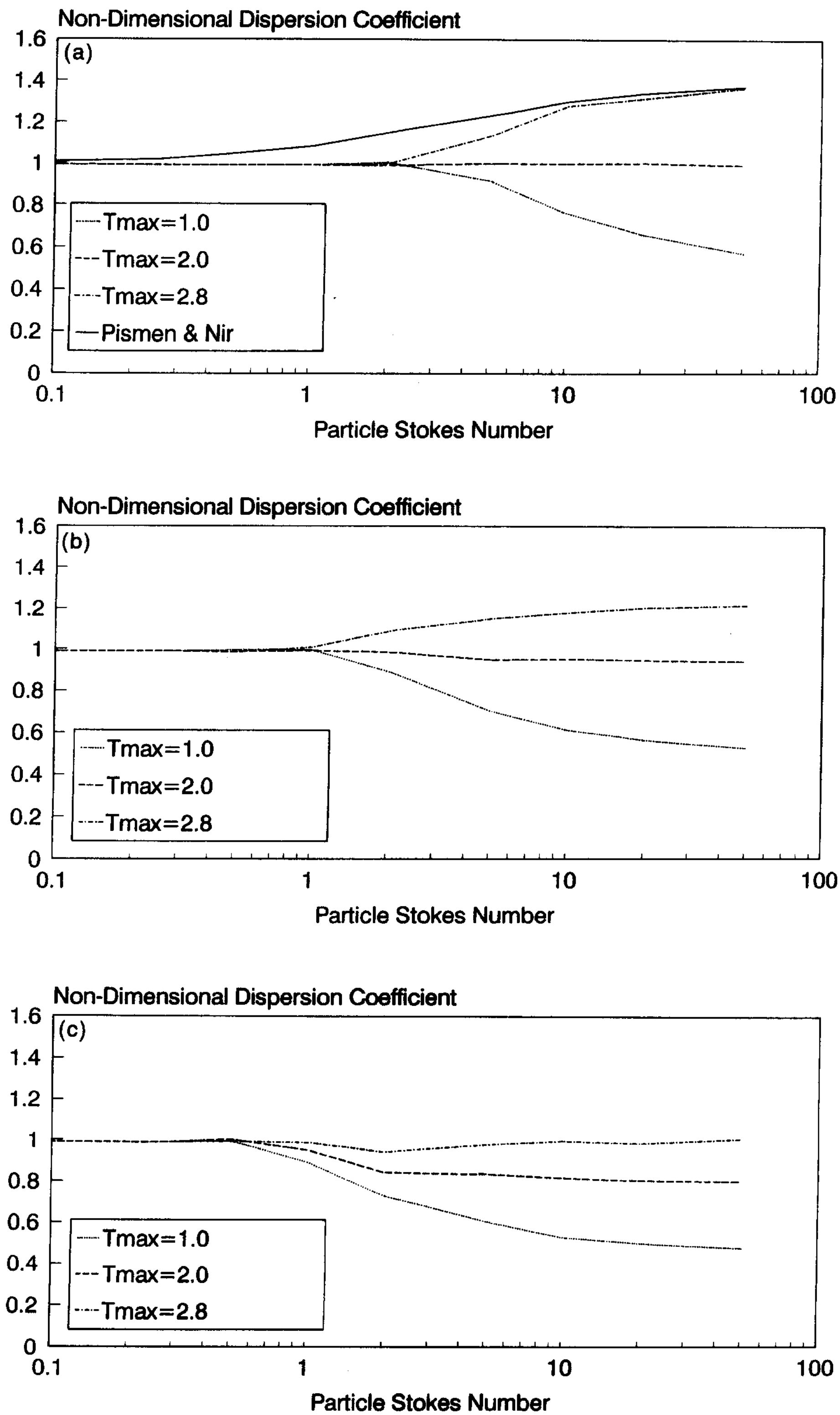

Figure 3. Long-time dispersion coefficients vs particle Stokes number for various T_{max}. (a) $\beta = 0.36$, (b) $\beta = 1.0$ and (c) $\beta = 2.0$.

It is clear from figure 3(a)–(c) that the computed dispersion coefficients are dependent upon both the Stokes number and the ratio τ_E/τ_L (which is determined in each case by the maximum interaction time). In addition, by comparing the results for a given value of T_{max} but with different values of β, the dispersion coefficients can also be seen to depend upon the turbulence structure parameter. For each value of β and T_{max}, the dispersion coefficient of fluid particles ($\tau_r \rightarrow 0$) is predicted correctly. This is an important feature since the method must certainly function properly for fluid particles, independently of the eddy length and the maximum interaction time. For a given value of β, the particle dispersion coefficient is equal to the dispersion coefficient of fluid particles over a range of τ_r/τ_L. Above a certain value of the Stokes number τ_r/τ_L (which appears from the results to be close to $1/\beta\tau_L$), the particles appear to lose their "fluid-like" quality and the particle dispersion coefficient differs from that of fluid particles.

In each case, the ratio $\bar{D}_p/\bar{D}_f$ for the heaviest particles ($\tau_r/\tau_L = 50$) is close to the ratio τ_E/τ_L found using figure 2. The case $T_{max} = 2\tau_L$, corresponds to the "standard" eddy interaction model where the maximum interaction time is equal to the fluid particle interaction time T_f. In this case, the dispersion coefficient for the heaviest particles is always slightly less than 1.

For the case $\beta = 0.36$, for this value of T_{max}, the relationship between dispersion coefficient and relaxation time follows qualitatively the same form as the results of Pismen & Nir (1978). The agreement with Pismen & Nir's results is excellent for fluid-like "low-inertia" particles ($\tau_r/\tau_L < 0.2$) and for "high-inertia" particles ($\tau_r/\tau_L > 10$). The dispersion coefficient for particles in the "mid-inertia" range are under-predicted compared with Pismen & Nir's results. However, even in this range, the predicted dispersion coefficients are always within approximately 15% of the analytical results. The simulations show that, by choosing the correct eddy length L_e, fluid particle interaction time T_f and maximum interaction time T_{max} so that the Eulerian longitudinal integral length scale Λ_E and the Eulerian and Lagrangian integral time scales (τ_E and τ_L, respectively) in the simulated turbulence are equal to those in the actual turbulence, the eddy interaction model can lead to very good predictions of particle dispersion.

By a suitable choice of $T_{max} > T_f$, the modified eddy interaction model can therefore be used in practice to predict enhanced dispersion of heavy particles. By choosing T_{max} to be less than T_f, the dispersion coefficient for very heavy particles is less than that for fluids. This is demonstrated in figure 3(a)–(c) for $T_{max} = \tau_L$, in which case the dispersion coefficient of the heaviest particles is only approximately half of the dispersion coefficient of fluid particles, dependent on the turbulence structure. It should be noted that any ratio $\bar{D}_p/\bar{D}_f$ for heavy particles could be chosen by choosing the value of T_{max} leading to the appropriate value of τ_E/τ_L.

4. CONCLUSIONS

Eulerian temporal auto-correlation functions and integral time scales in eddy interaction models have been evaluated using numerical integration for several combinations of turbulence structure parameter $\beta = u'\tau_L/\Lambda_E$ and maximum interaction time T_{max}.

(i) By suitable choice of T_{max} (which may be greater than the fluid particle interaction time T_f, allowing non-fluid particles to interact with eddies for longer than fluid particles are allowed) it is possible to predetermine the ratio between the Eulerian and Lagrangian integral time scales. The value of T_{max} required for a given ratio τ_E/τ_L has been shown to depend on the turbulence structure.

(ii) By choosing the correct eddy length L_e, fluid particle interaction time T_f and maximum interaction time T_{max} so that the Eulerian longitudinal integral length scale and the Eulerian and Lagrangian integral time scales in simulations of turbulent flows are equal to those in the actual turbulence, the modified eddy interaction model can lead to very good predictions of particle dispersion.

(iii) Using a suitably large maximum interaction time, it is possible to ensure that heavy particles disperse more rapidly, in the long term, than fluid particles. Use of a suitably small T_{max} ensures that fluid particles disperse more rapidly than heavy particles.

Acknowledgements—The author is indebted to Dr P. W. James for many helpful suggestions and discussions. This work has been funded under SERC Grant No. GR/H42 437.

REFERENCES

Deutsch, E. 1992 Dispersion de particules dans une turbulence homogene isotrope stationnaire calculee par simulation numerique des grand echelles. Ph.D. thesis, Ecole Centrale de Lyon, France.

Gosman, A. D. & Ioannides, E. 1981 Aspects of computer simulation of liquid-fuelled combustors. Paper AIAA-81-0323, presented at *AIAA 19th Aerospace Sciences Meeting*, St Louis, MO.

Govan, A. H., Hewitt, G. F. & Ngan, C. F. 1989 Particle motion in a turbulent pipe flow. *Int. J. Multiphase Flow* **15,** 471–481.

Graham, D. I. & James, P. W. 1996 Turbulent dispersion of particles using eddy interaction models. *Int. J. Multiphase Flow* **22,** 157–185.

Hutchinson, P., Hewitt, G. F. & Dukler, A. E. 1971 Deposition of liquid or solid dispersions from turbulent gas streams: a stochastic model. *Chem. Engng Sci.* **26,** 419–439.

Kraichnan, R. H. 1971 Diffusion by a random velocity field. *Phys. Fluids* **11,** 22–31.

Minier, J. P. & Pozorski, J. 1992 Analysis of existing Lagrangian models and new propositions for particle dispersion in homogeneous stationary turbulence. Report HE-44/92.29, Electricite de France, France.

Pismen, L. M. & Nir, A. 1978 On the motion of suspended particles in stationary homogeneous turbulence. *J. Fluid Mech.* **84,** 193–206.

Reeks, M. W. 1977 On the dispersion of small particles suspended in an isotropic turbulent fluid. *J. Fluid Mech.* **83,** 529–546.

Shuen, J.-S., Solomon, A. S. P., Zhang, Q.-F. & Faeth, G. M. 1983 A theoretical and experimental study of turbulent particle-laden jets. Report CR-168293, NASA, U.S.A.

Squires, K. D. & Eaton, J. K. 1991 Measurements of particle dispersion obtained from direct numerical simulation of isotropic turbulence. *J. Fluid Mech.* **226,** 1–35.

Wang, L.-P. & Stock, D. E. 1992 Stochastic trajectory models for turbulent diffusion: Monte Carlo process versus Markov chains. *Atmospher. Envir.* **26A,** 1599–1607.

Wells, M. R. & Stock, D. E. 1983 The effects of crossing trajectories on the dispersion of particles in a turbulent flow. *J. Fluid Mech.* **136,** 31–62.

Yudine, M. I. 1959 Physical considerations on heavy particle diffusion. *Adv. Geophys.* **6,** 185–191.